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Modelling optical bistability with hybrid silicon-plasmonic resonators

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Scope

- **Nonlinear control in guided-wave plasmonics**
  - ☑ Sub-λ confinement
  - ☒ Resistive losses

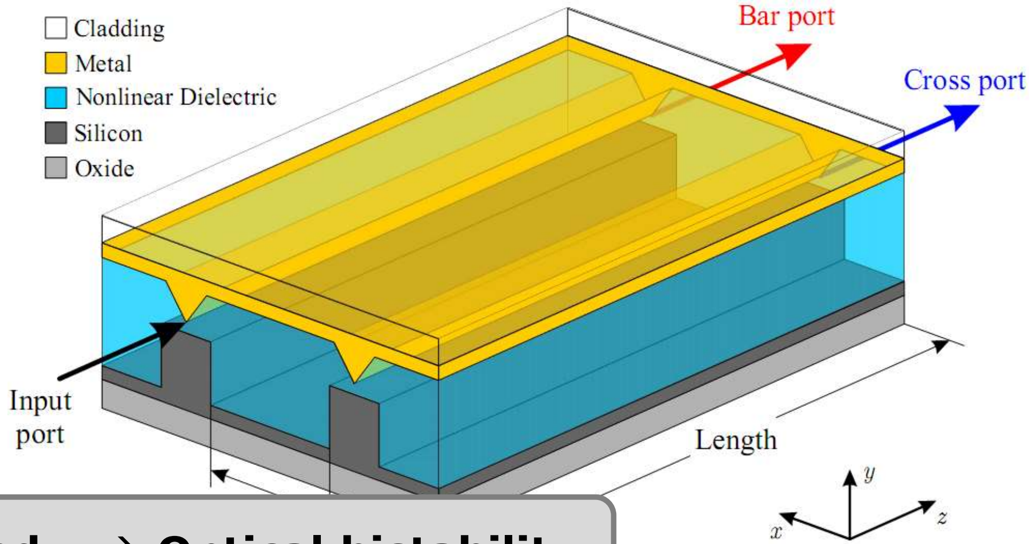
→ *Hybrid plasmonic waveguides*
(best compromise)

[*Oulton, Nat. Photon. 2, 2008*]
[*Wu, Opt. Express 18, 2010*]

- **Ultrafast phenomena**
  - ☑ Kerr (third-order susceptibility)
  - ☑ Free carrier dispersion (w/ carrier sweeping)

- **Nonlinear directional coupler**
  - ☒ P_{th} > 10s or even 100s W **!**
  (loss → short interaction length)

[*Milián, APL 98, 2011*]
[*Kriesch, CLEO/QELS 2012*]
[*Pitilakis, JOSA B 30, 2013*]



Resonator enhanced… → Optical bistability

Presentation outline

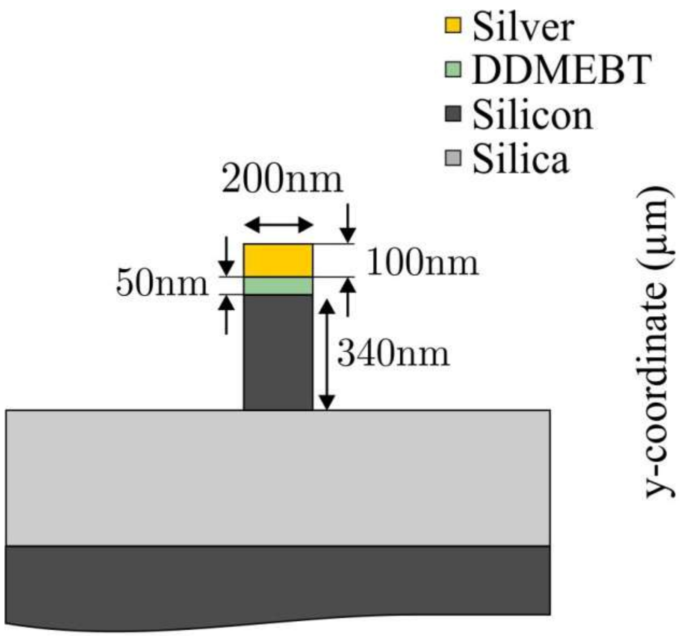
- **Nonlinear Travelling-Wave Resonator Structure**
  - Physical system: Side-coupled disk
  - Perturbation Theory & CMT Framework
  - Effect of Model Parameters on Bistability Curve
  - System Design
  - CW Performance Assessment
  - Temporal response

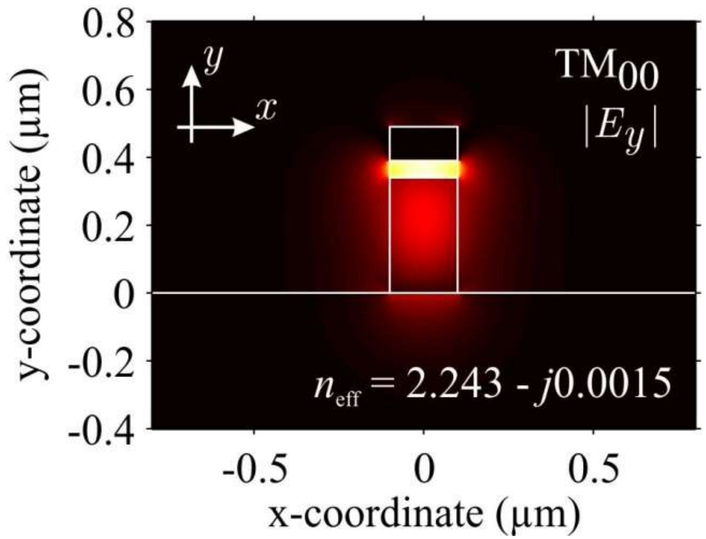
- **Nonlinear Standing-Wave Resonator Structure**
  - Physical system: Side-coupled Bragg resonator
  - System Design
  - CW Performance Assessment
  - Temporal response

Nonlinear hybrid plasmonic waveguide (NLCGS)

- CGS with nonlinear polymer **DDMEBT** (n_2 = 1.7 × 10^{-17} m^2/W)
  - **Silver** for lower resistive losses

Material Properties
[*Esembeson, Adv. Mater. 20, 2008*]
[*Koos, Nat. Photon. 3, 2009*]
[*Johnson, PRB 6, 1972*]





- ☑ Strong confinement: $\boldsymbol{A_{eff}}$ **~ 0.05 μm²**
- ☑ High nonlinear coefficient: $\boldsymbol{\gamma_{wg}}$ **~ 1500 W⁻¹m⁻¹**
- ☒ L_{prop} ~ 80 μm → **Small effective length**
- ☒ **44 W** for directional coupler switching → … resonator enhanced

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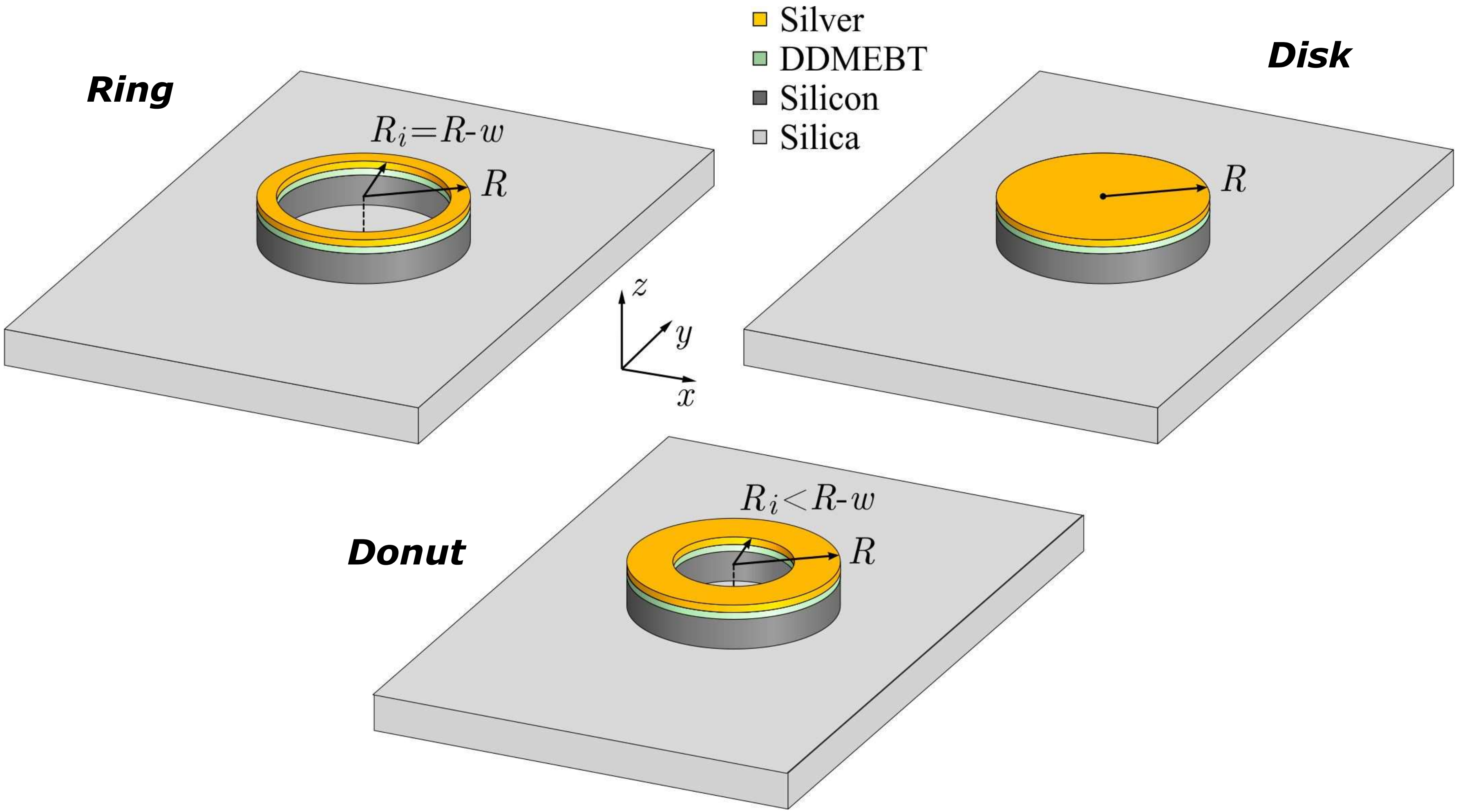
NLCGS-based travelling-wave resonators

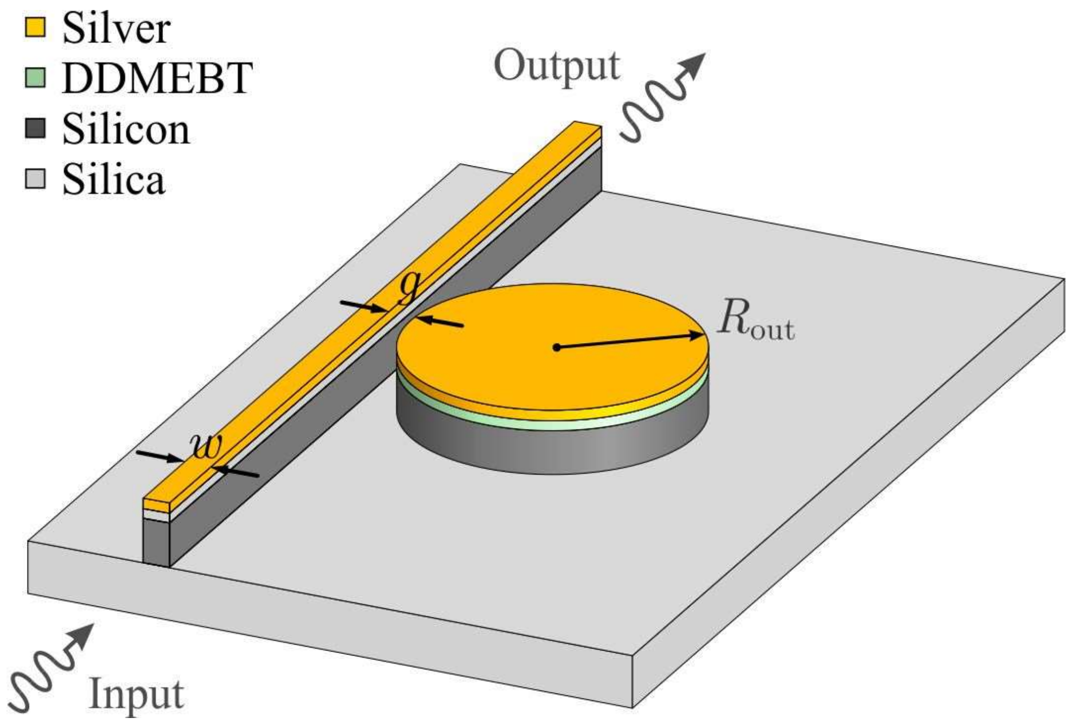


- *Disk*: **Reduced radiation losses** compared to ring
- *Donut*: For **suppressing unwanted modes** of the disk

Nonlinear disk resonator structure

Nonlinear disk side-coupled to CGS bus waveguide

- Disk: Reduced radiation losses → Higher *Q*
- **Intensity build-up** in resonator → Nonlinearity enhancement
- Compact structure



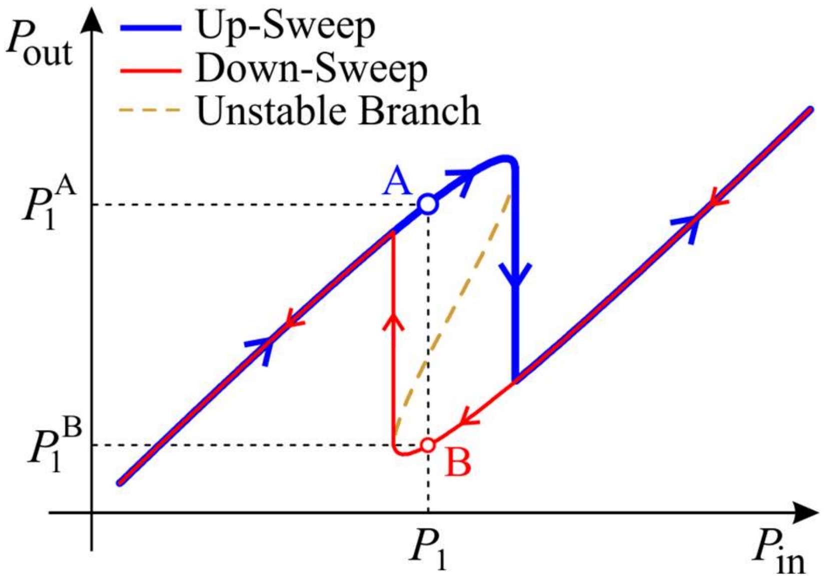
[*Tsilipakos, JOSA B 31, 1698, 2014*]

$\chi^{(3)}$ → $\Delta n \propto |E|^2$

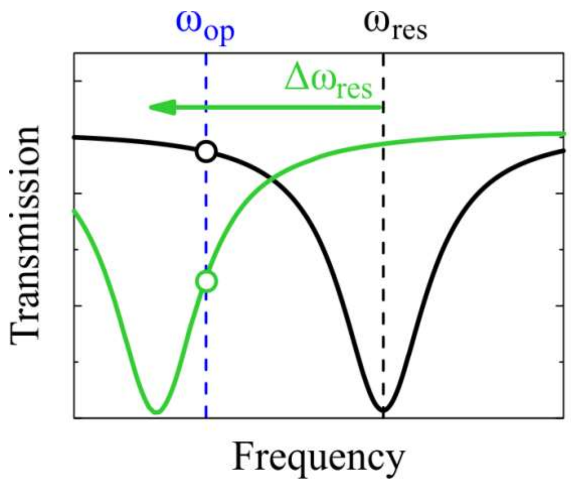
ω_{res} shifts !
($\Delta\omega<0$ for $n_2>0$)

Nonlinear disk resonator structure: Optical bistability

Nonlinearity + optical feedback
→ Hysteresis loop (bistability)



Prerequisite:
$\omega_{op} < \omega_{res}$ since $\Delta\omega_{res} < 0$

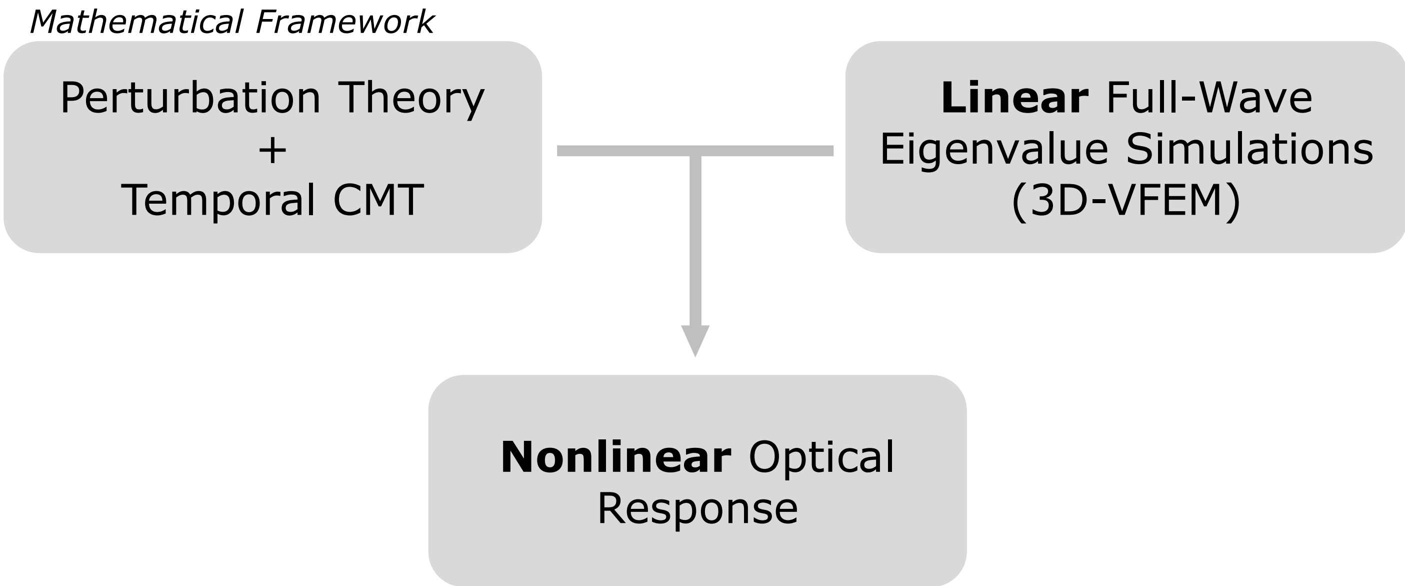


Increase in transmission

"Lorentzian" dependence of transmission on $\Delta\omega_{res}$ → hysteresis loop

Modelling Framework

Modelling Framework



[*Bravo-Abad, JLT 25, 2539, 2007*]

Perturbation Theory

Uncoupled nonlinear resonator

- *Linear regime*: Unperturbed resonant frequency ω_0
- *Nonlinear regime*: **Frequency shift Δω** due to **nonlinear self-action**

$$\frac{\Delta\omega}{\omega_0} = -c_0 \left(\frac{\omega_0}{c_0}\right)^3 \kappa\, n_2^{\max}\, W$$

[*Bravo-Abad, JLT 25, 2007*]

- Proportional to stored energy *W*
- Proportional to **nonlinear feedback parameter** κ ($\propto 1/V_{\text{eff}}$)
  - Measure of mode overlap w/ nonlinear material

$$\kappa = \left(\frac{c_0}{\omega_0}\right)^3 \frac{\iiint_V \frac{1}{3} n_2(\mathbf{r}) n^2(\mathbf{r}) \left[\left(\mathbf{E}_0 \cdot \mathbf{E}_0\right)\left(\mathbf{E}_0^* \cdot \mathbf{E}_0^*\right) + 2\left|\mathbf{E}_0\right|^4\right] \mathrm{d}V}{\left[\iiint_V n^2(\mathbf{r}) \mathbf{E}_0 \cdot \mathbf{E}_0^* \, \mathrm{d}V\right]^2 n_2^{\max}}$$

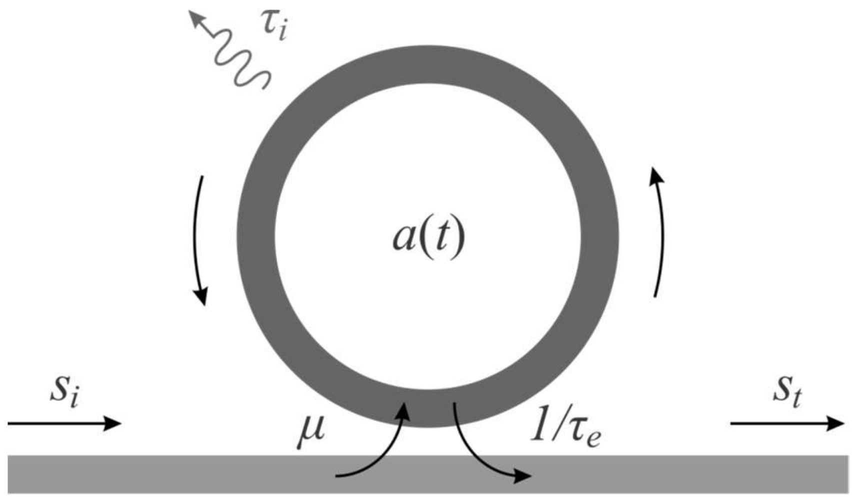
- **Redshift** ($\Delta\omega < 0$) for $n_2 > 0$
- From **linear** full-wave simulation (3D-VFEM)

Temporal coupled mode theory (CMT)

$$\frac{da}{dt} = j\left(\omega_0 + \Delta\omega\right)a - \frac{1}{\tau_i}a - \frac{1}{\tau_e}a + \mu s_i$$

$a(t)$	cavity amplitude, $\|a(t)\|^2=W$
ω_0	unperturbed resonant frequency
$\Delta\omega$	nonlinear frequency shift
τ	photon lifetime, $\tau=2Q/\omega$
μ	coupling coefficient, $\mu=(2/\tau_e)^{1/2}$
s	w/g mode amplitudes, $\|s\|^2=P$

$$s_t = s_i + \mu a$$



- **Steady-state response**

$$T \equiv \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{(1-r_Q)^2 + (\bar{\delta} - \tau_i\Delta\omega)^2}{(1+r_Q)^2 + (\bar{\delta} - \tau_i\Delta\omega)^2}$$

➢ *Cannot construct loop since $\Delta\omega \propto W$!*

Quality factor ratio

$$r_Q = Q_i \,/\, Q_e = \tau_i \,/\, \tau_e$$

Normalized detuning

$$\bar{\delta} = \tau_i(\omega - \omega_0)$$

Closed-form relation for CW nonlinear response

$$W = Q_i \frac{P_{\text{in}} - P_{\text{out}}}{\omega_0}$$

$$\frac{\Delta\omega}{\omega_0} = -c_0 \left(\frac{\omega_0}{c_0}\right)^3 \kappa\, n_2^{\max} W$$

$$\tau_i \Delta\omega = -\frac{P_{\text{in}} - P_{\text{out}}}{P_0}$$

System characteristic power

$$P_0 = \frac{1}{2\left(\frac{\omega_0}{c_0}\right)^2 \kappa Q_i^2 n_2^{\max}} \propto \frac{1}{\kappa Q_i^2}$$

$$T \equiv \frac{p_{\text{out}}}{p_{\text{in}}} = \frac{(1 - r_Q)^2 + (\bar{\delta} + p_{\text{in}} - p_{\text{out}})^2}{(1 + r_Q)^2 + (\bar{\delta} + p_{\text{in}} - p_{\text{out}})^2}$$

[*Tsilipakos, JOSA B 31, 2014*]

- **Closed-form** relation
- Allows for constructing the hysteresis loop
- Admits **three** real solutions (for appropriate p_{in} levels)
- **Detuning threshold:**

$$\bar{\delta} < \bar{\delta}_{\text{th}} = -(1 + r_Q)\sqrt{3}$$

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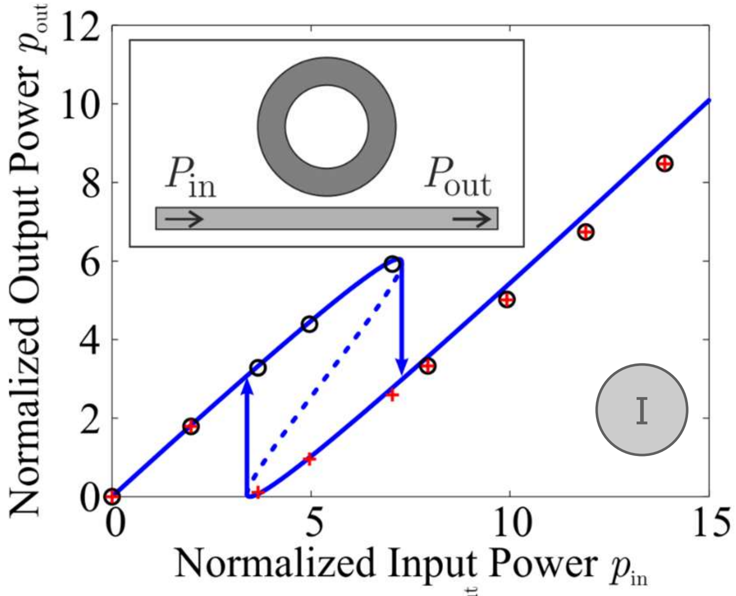
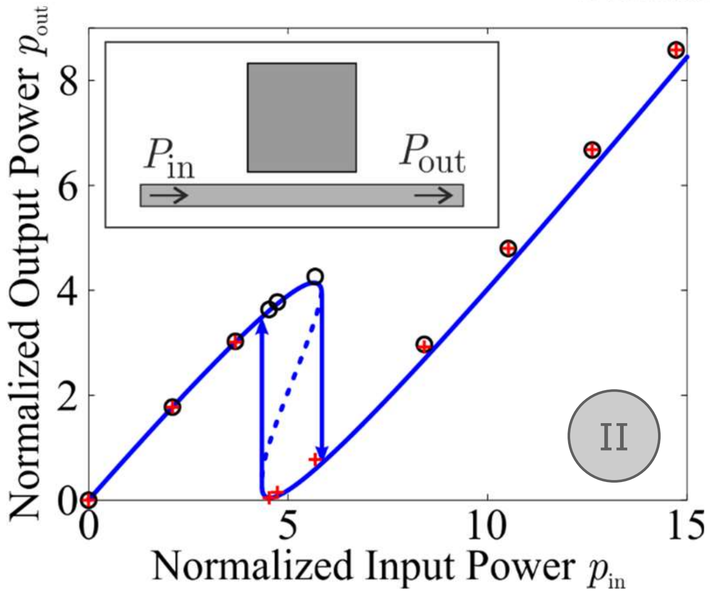
Comparison with full-wave nonlinear simulation (CW)

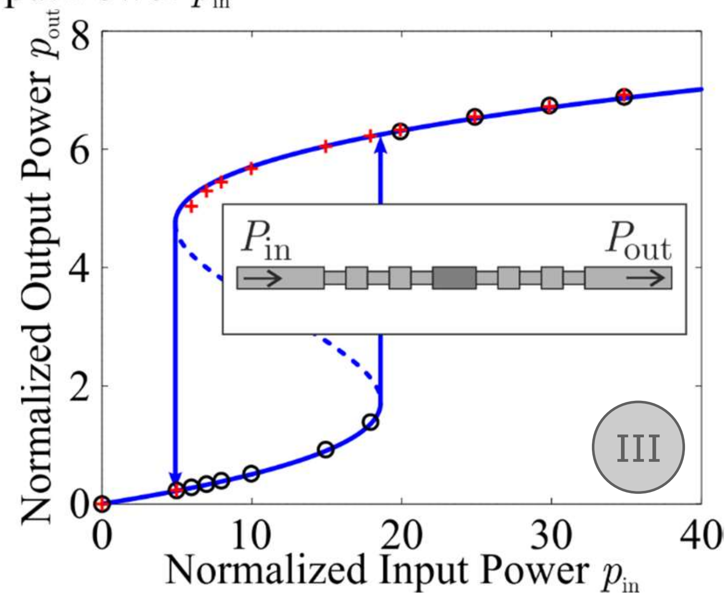
2D Si Structures

I. TW resonator
II. SW, side coupling
III. SW, direct coupling



- CMT + Linear FEM
- NL-FEM, upsweep
- NL-FEM, downsweep

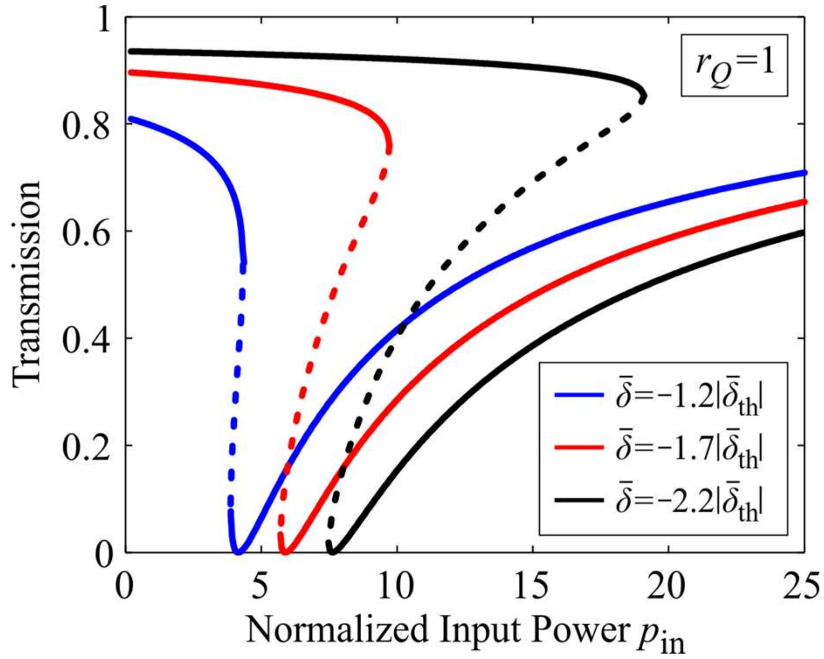




Effect of Model Parameters

Detuning: Effect on bistability curve

- ❑ r_Q = 1 (critical coupling) | Same trends for any r_Q value
- ❑ **δ = {-1.2|δ_{th}|, -1.7|δ_{th}|, -2.2|δ_{th}|}**

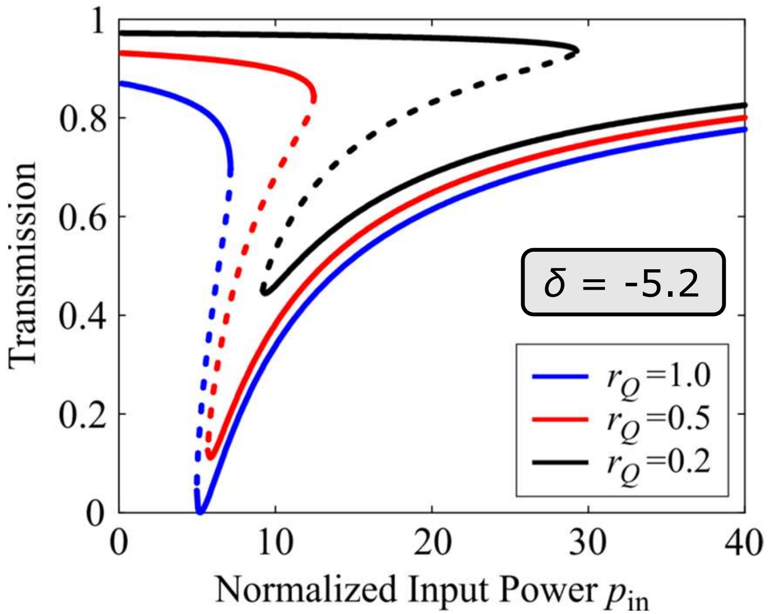


Increase in |δ|

- ☒ **Higher input power required**
- ❑ Loop span increases
- ☑ Higher maximum transmission

Quality factor ratio: Effect on bistability curve

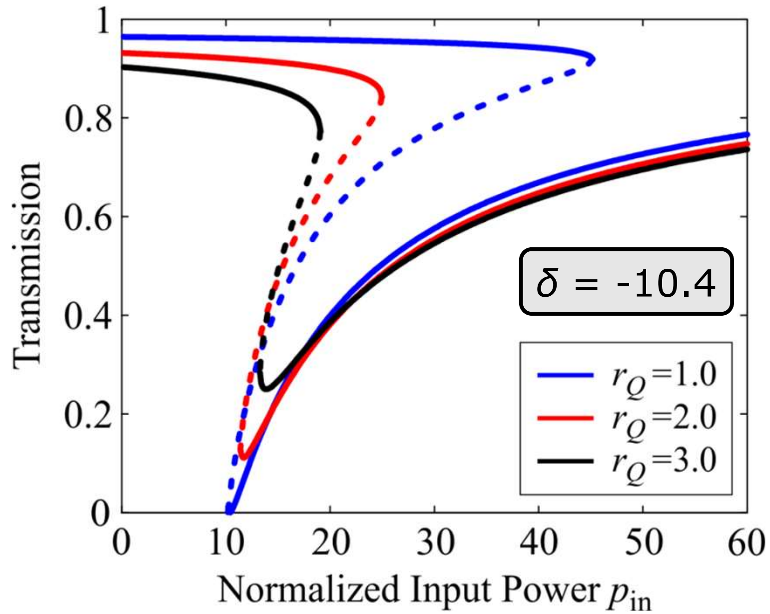
❑ **r_Q < 1 regime** {1.0, 0.5, 0.2}



Decreasing r_Q below 1:

- ☒ Higher input power required
- ☒ **T_{min} increases (loop elevation)**
- ❑ Loop span increases
- ☑ Higher T_{max}

❑ **r_Q > 1 regime** {1.0, 2.0, 3.0}



Increasing r_Q above 1:

- ☒ Higher input power required
- ☒ **T_{min} increases (loop elevation)**
- ❑ Loop span decreases
- ☒ Lower T_{max}

To recapitulate…

Optimum parameters – Design specifications

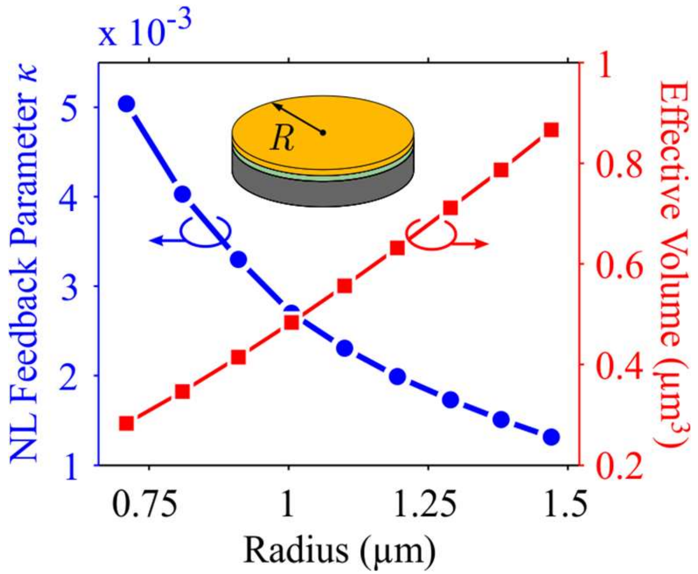
- ❑ Detuning close to respective threshold
  - Loop appears for lower input power
- ❑ Critical coupling ($r_Q = 1$)
  - ER between bistable states is infinite
  - Deviating from $r_Q = 1 \rightarrow$ ER degradation
- ❑ Minimum characteristic power P_0

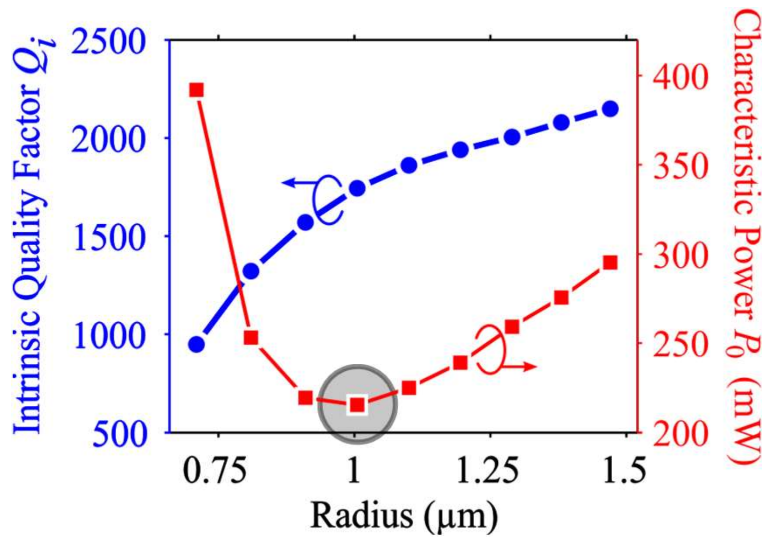
Physical System Design

The uncoupled disk as an eigenvalue problem: P_0 minimization

Parametric analysis w.r.t. radius *R*

- $R < 0.7$ → Significant rad. losses | $R > 1.5$ → Q bound by res. losses
- Each marker corresponds to different azimuthal order
- **Minimum P_0** or maximum κQ_i^2 product
- κ, Q_i : **Opposing trends** with radius



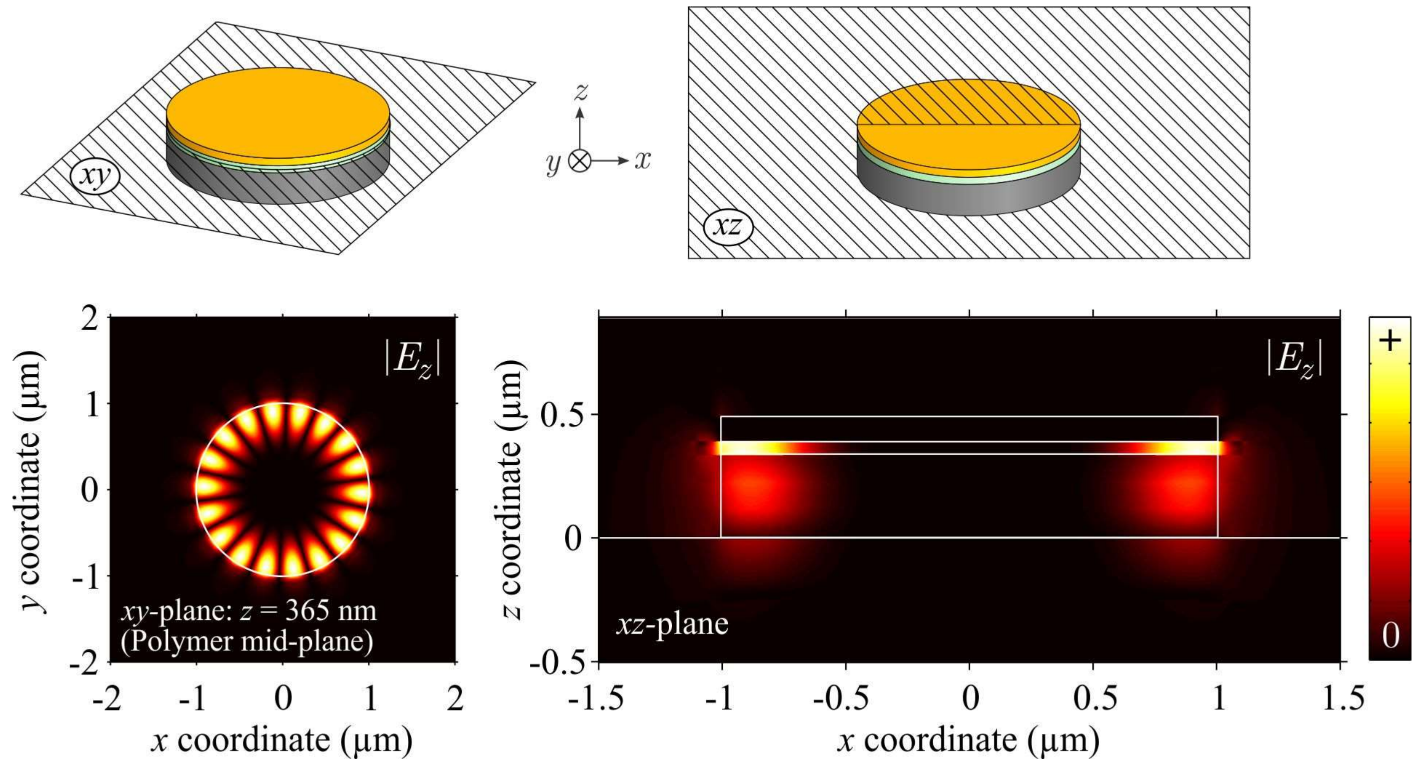


Optimum value: *R* = 1 μm

- $\kappa = 2.7\times10^{-3}$ | $Q_i = 1750$
- **P_0 = 215 mW**

The uncoupled disk as an eigenvalue problem: Resonant mode

- **Resonant mode** (R = 1 μm, m = 9, λ_{res} = 1553 nm)

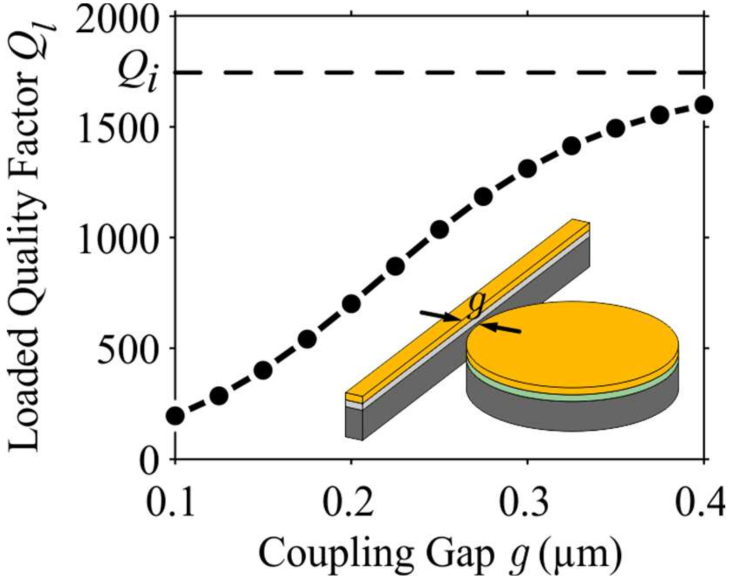


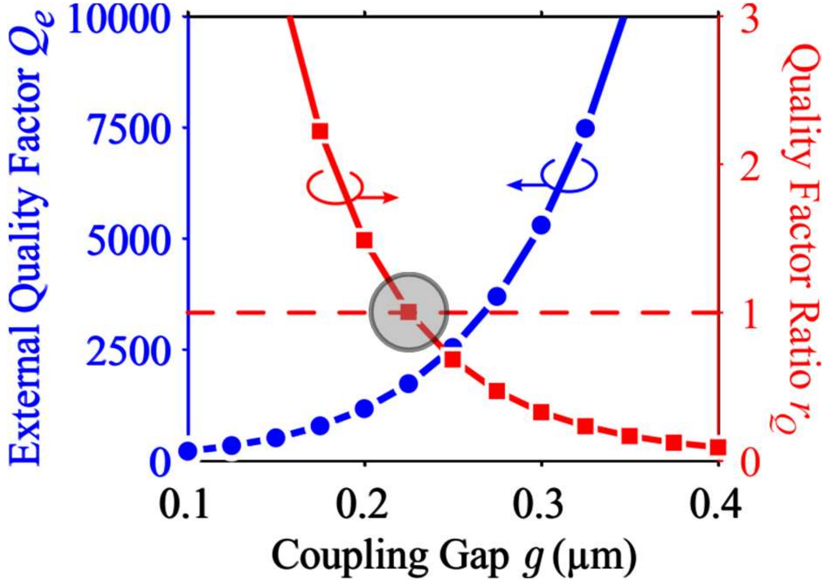
- Strong confinement: $\boldsymbol{V_{eff}}$ **~ 0.5 μm³**
- Excellent **overlap** w/ nonlinear material
- Plasmonic nature (peaks at interface)

The coupled disk as an eigenvalue problem: Critical coupling

Parametric analysis w.r.t. coupling gap *g*

- **Loaded** quality factor Q_l
- **External** quality factor: $Q_e^{-1} = Q_l^{-1} - Q_i^{-1}$
- Quality factor **ratio** $r_Q = Q_i / Q_e$





Critical coupling (r_Q=1) → ***g* = 225 nm**

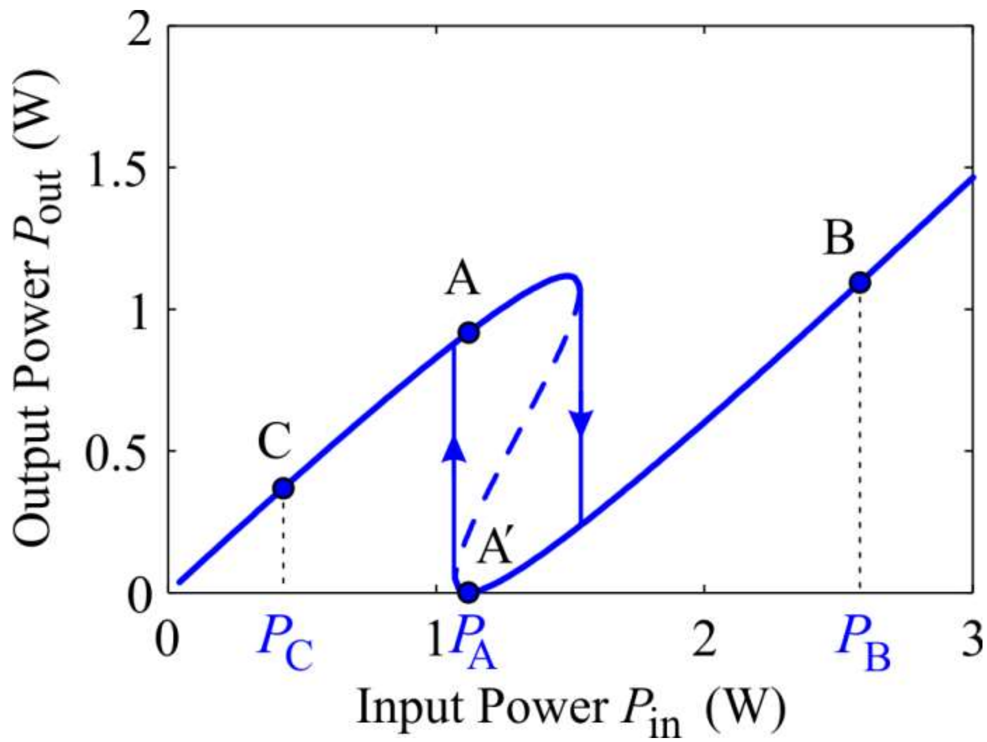
To recapitulate…

Physical system design

- min{P_0} → ***R* = 1 μm**
- r_Q = 1 → ***g* = 225 nm**
- δ = -5.2 (1.5δ_{th}) → **λ = 1555.3 nm**
  (2.3 nm above λ_0 = 1553 nm)

Performance Assessment

Bistability curve



Physical system

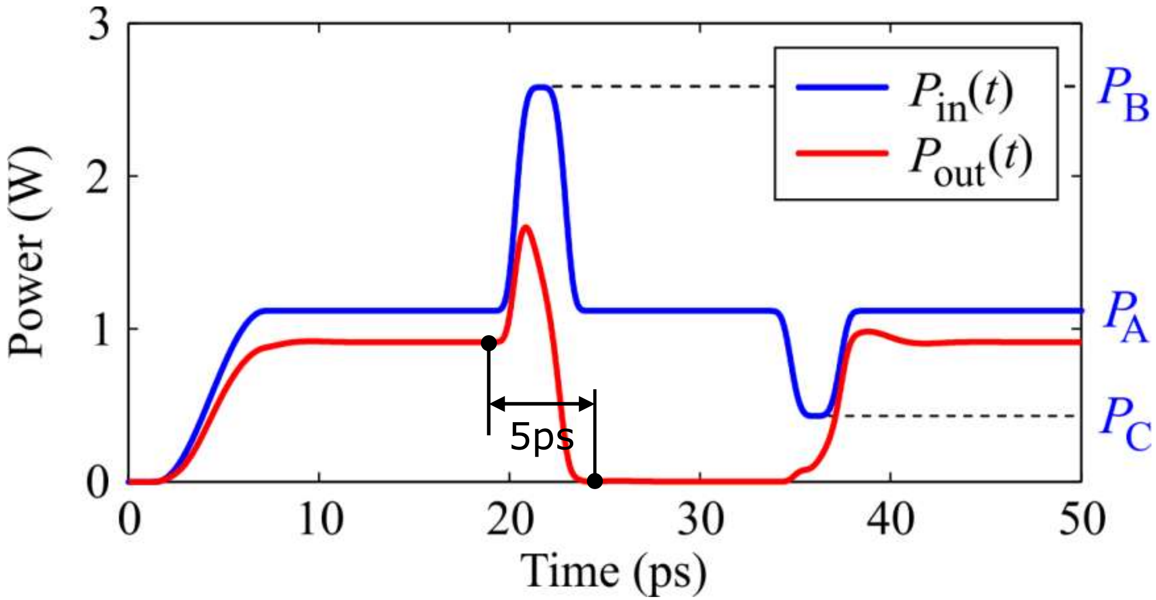
- ❑ ***R* = 1 μm**
- ❑ ***g* = 225 nm**
- ❑ ***λ* = 1555.3 nm**

Performance

- ❑ ***P*$_{in}$ ~ 1W** for bistability (P_A = 1.12 W)
- ❑ **ER → ∞** @ P_A
- ❑ IL ~ 1 dB @ point A
- ❑ Points B, C for toggling between bistable states A, A′

Temporal response

- Initially $P_{in} = P_A$ → System at **high-output** state
- 2^{nd}-order **super-Gaussian pulses** (FWHM = 2.25 ps) for **toggling state**
  - 1^{st} pulse (peak) → *ABA′* → **low-output** state
  - 2^{nd} pulse (dip) → *A′CA* → **high-output** state



☑ **5 ps** (~3τ_l) for settling at new state
☑ Toggling frequency up to **100 GHz!**

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- **Nonlinear Standing-Wave Resonator Structure**
  - Physical system: Side-coupled Bragg resonator
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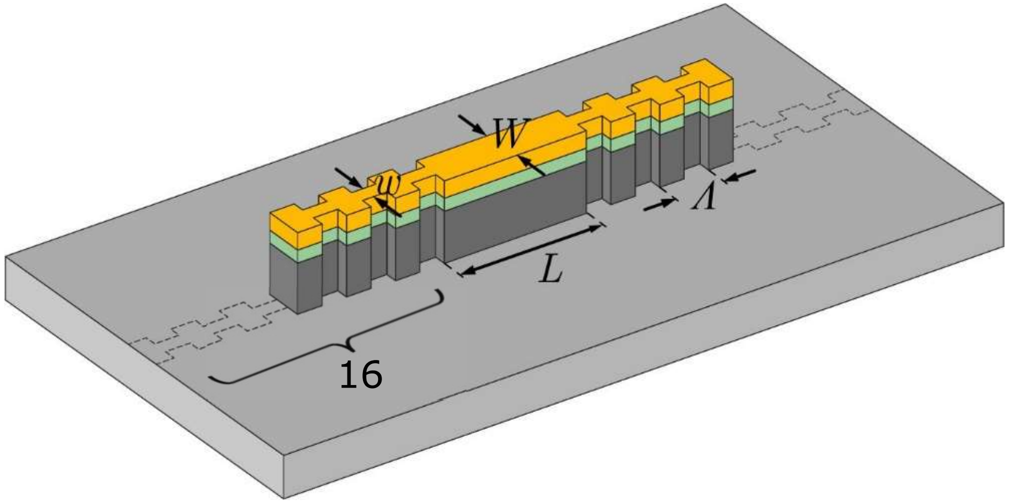
Nonlinear Bragg grating resonator

Bragg grating resonator

- ❑ **16-period** reflectors (~5μm long)
  (reflectivity-compactness compromise)
- ❑ Design procedure
  - Specify (W, w, L) parameters
  - Optimum performance
    (min{P_0}, max{ER})



CMT framework

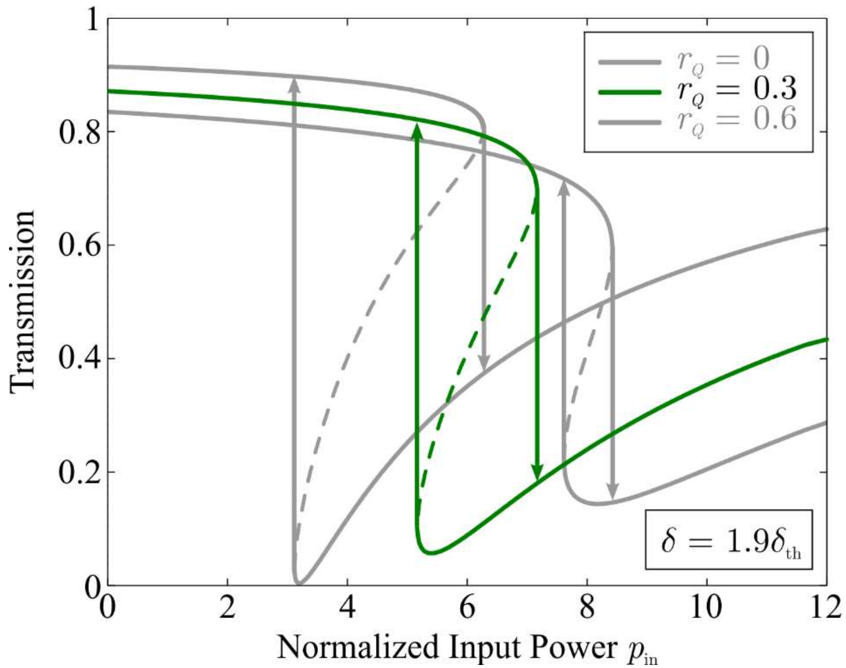
- ❑ Slightly different r_Q, P_0 definitions

$$r_Q = Q_e / Q_i \qquad P_0 \propto (\kappa Q_e^2)^{-1}$$

Hysterisis loop

- ❑ r_Q → 0 for high ER
- ❑ But P_0 increases …

r_Q**=0.3** (ER>10dB)

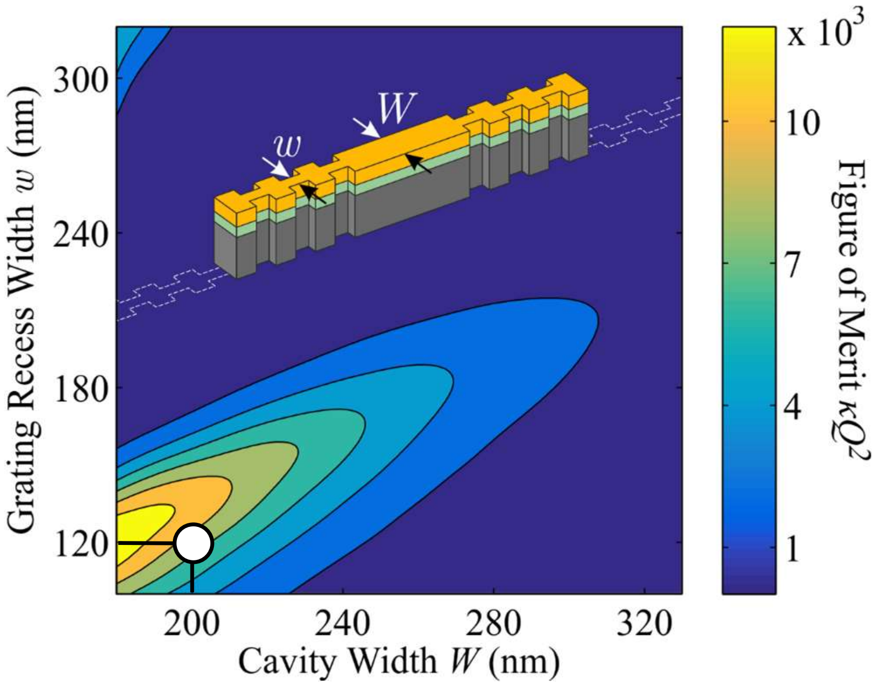


Physical System Design

The uncoupled resonator

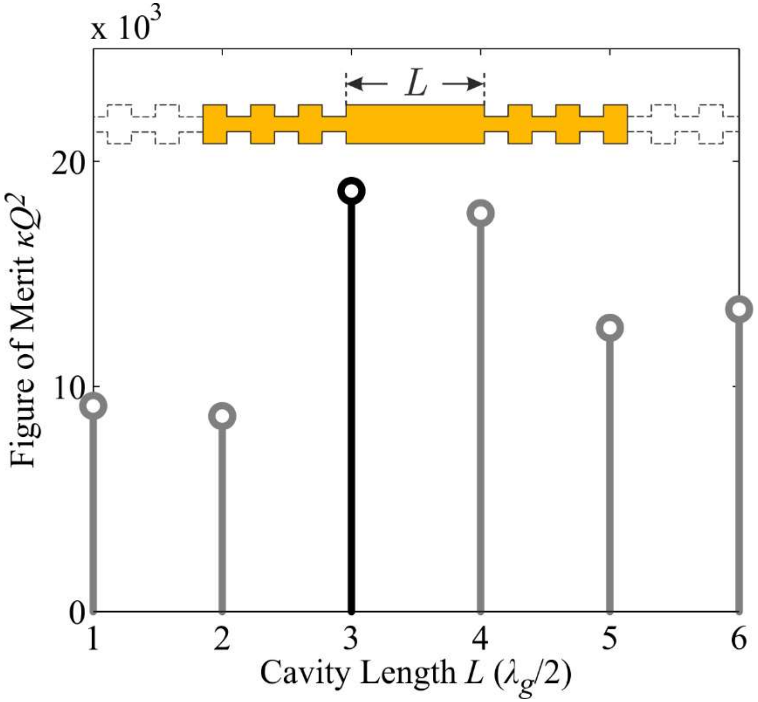
Width engineering

- ❑ $L=\lambda_g/2$
- ❑ Narrow cavity widths
  - ▪ W < 250nm
- ❑ **(W, w) = (200nm, 120nm)**



Length engineering

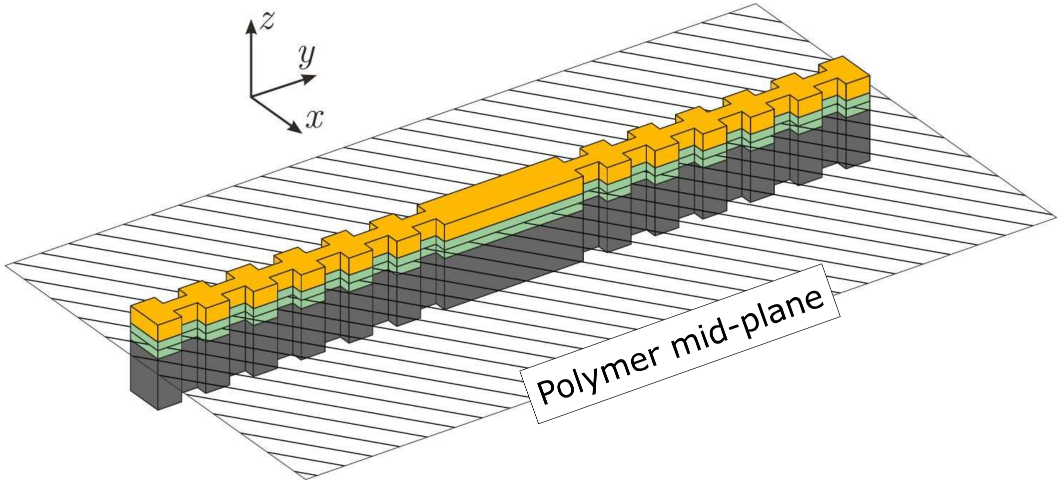
- ❑ Third-order mode
- ❑ Reduced rad. losses
  (const res. losses)
- ❑ **L ~ 1μm ($3\lambda_g/2$)**

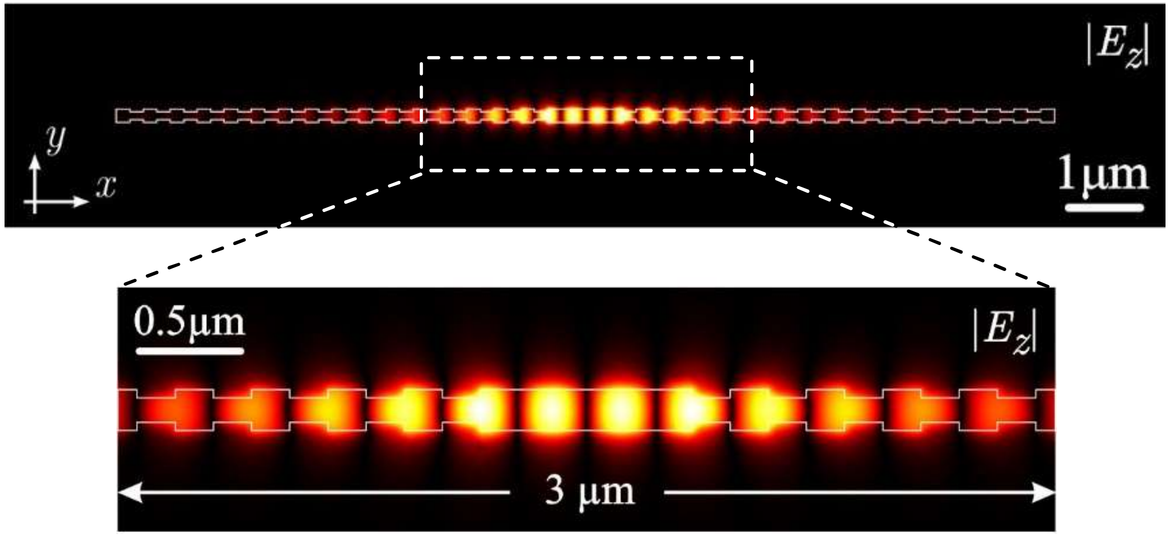


The uncoupled resonator: Resonant mode

Resonant mode

- m = 3, λ_{res} = 1550 nm
- V_{eff} = 0.1 μm^3
- Q_i = 500
  - limited by Q_{res} = 560
- P_0 ~ 1.5 W (r_Q=0.3) (x6)
  - κ = 2.5×10^{-2} (x10)
  - Q_e = 150 (÷11)

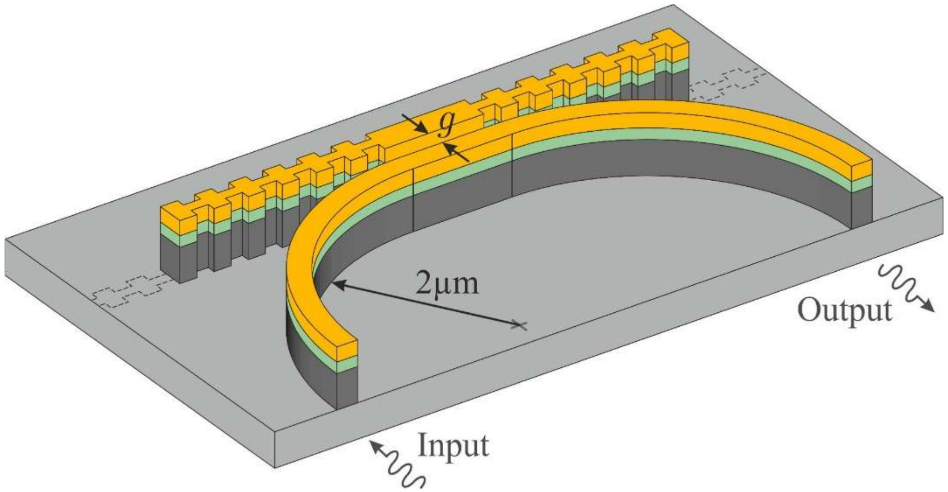


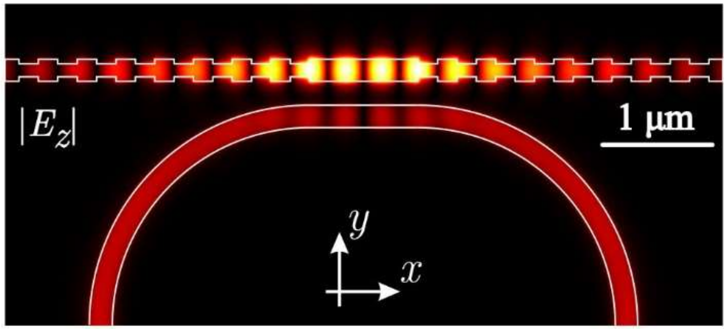


Coupled resonator

Coupled Bragg resonator

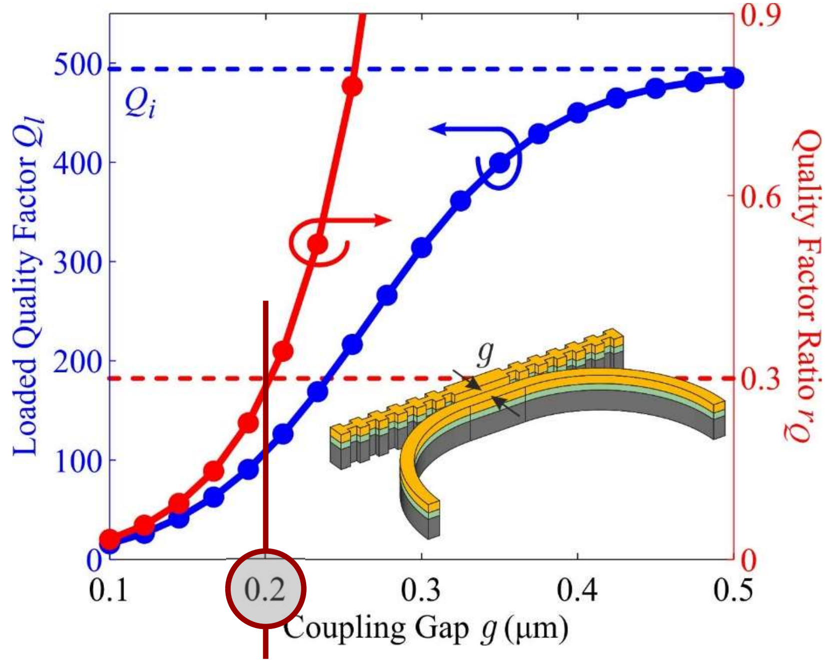
- Coupling gap g
- **Curved** access waveguide
- 2-μm radius quadrants





Parametric analysis w.r.t. gap *g*

- **Loaded** quality factor Q_l
- **External** quality factor: $Q_e^{-1} = Q_l^{-1} - Q_i^{-1}$
- Quality factor **ratio** $r_Q = Q_e / Q_i$



r_Q=0.3 → ***g* = 200 nm**

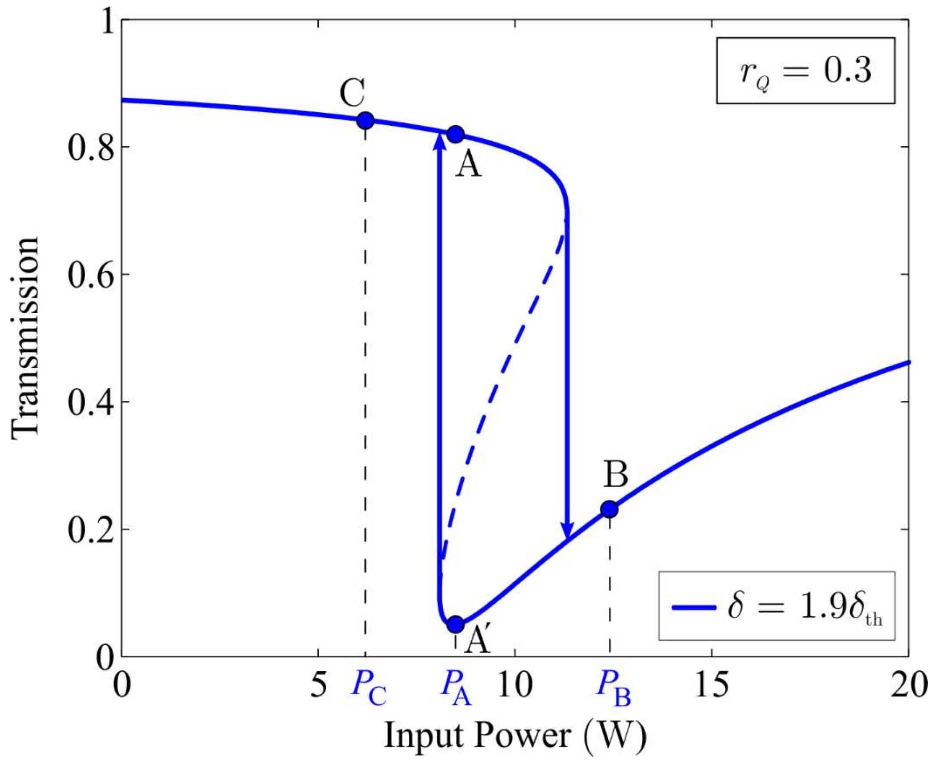
To recapitulate…

Physical system design

- min{P_0} → { ***W* = 200 nm**, ***w* = 120 nm**, ***L* = 1 μm** }
- r_Q = 0.3 → ***g* = 200 nm**

Performance Assessment

Bistability curve



Physical system

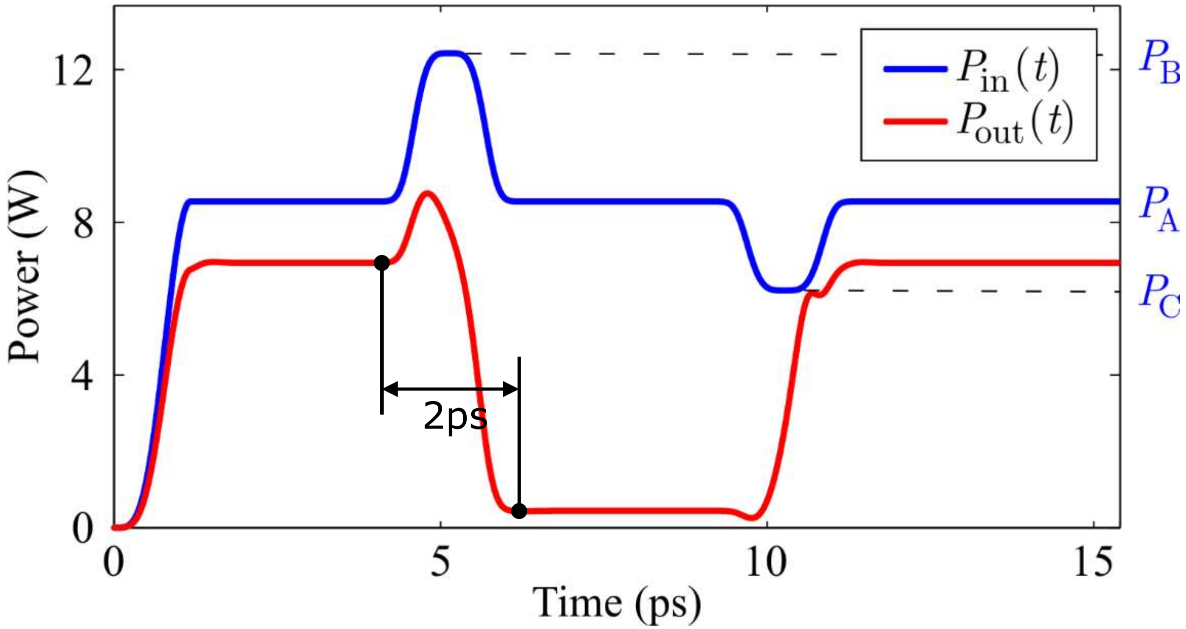
- ***L*** **= 1 μm**
- ***W*** **= 200 nm**
- ***w*** **= 120 nm**
- ***g*** **= 200 nm**

Performance

- **P_{in} ~ 8W** for bistability
- **ER ~ 12 dB** @ P_A
- IL < 1 dB @ point A
- Points B, C for toggling between bistable states A, A′

Temporal response

- 2nd-order **super-Gaussian pulses** (T_0 = 0.5 ps) for **toggling state**
  - 1st pulse (peak) → *ABA′* → **low-output** state
  - 2nd pulse (dip) → *A′CA* → **high-output** state



☑ **2 ps** for settling at new state
☑ Toggling frequency > **100 GHz!**

Conclusion

Summary

- **Practical** plasmonic components for Kerr bistability
  - Travelling- or standing-wave resonator implementations
- **Reduced** power requirements w.r.t. directional coupler approach
- **High extinction ratio** (side coupling)
- **Ultrafast response**

To probe further …

- Impact of **free-carrier effects** (Si layer)
  - Important, despite the limited mode overlap with Si
  - Dominate over Kerr in CW ($\Delta\omega^{FCD}$ >> $\Delta\omega^{Kerr}$ & strong FCA)
  - Pulsed regime: sweeping for suppressing FCA
- FCD bistability
- Thermal bistability
  - TPA, FCA, Joule heating

Thank you!

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